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Total Beta: **A View from Outside**

By Dietmar Ernst and Werner Gleißner

For an observer from outside the U.S., the debate over total beta (TB) is quite extraordinary. Although the valuation of private companies is also discussed elsewhere, there is no "battle of the beta"[1] in comparable terms. The purpose of this article is to summarize the discussion of TB in the U.S. and to examine its various arguments in light of the recent professional and academic literature on TB in the German-speaking countries.

We aim to show how, by incorporating semi-investment theory valuation concepts and their replication methods, the TB approach can be integrated in such a way that it serves as a possible approach for capturing different degrees of diversification of business owners. We show that the results of a valuation based on the capital asset pricing model (CAPM) and TB represent special cases of a general valuation concept. Under this approach, existing financing restrictions can also be taken into account and the cost of equity can be derived from the credit rating or insolvency probability (rating-related model or risk coverage approach).

In the process, we demonstrate that the German literature has more in common with U.S. contributions that are not at the center of the current debate and that its recommendations for the implementation of the TB concept lead to a different approach than that of Butler and Pinkerton and their successors. We also discuss special heuristics that are based on the TB concept.

Our general belief concerning this approach is that the theoretical foundations for the valuation of private companies are much weaker than those for publicly listed corporations with well-diversified shareholders. Therefore, we should be modest and view our conclusions with an appropriate degree of skepticism. Nevertheless, as practical problems need to be solved in a timely manner, the practitioner cannot wait until theory delivers a sound tool someday. As practitioners, we seek to find the best available methods that can be used as a guideline to judge our work.

Conceptual Issues

In our view, there were two sources for the success of TB: the creation and naming of the approach by Aswath Damodaran in 2001,[2] and the contributions of Peter Butler and Keith Pinkerton, beginning in 2006,[3] including their development of the "Butler-Pinkerton Calculator" (BPC).

Damodaran's idea was quite simple: because the owners of private companies typically are not diversified, the risk-reducing effect of diversification should be eliminated from the estimation of the risk premium. As the prevailing model, the CAPM captures this effect through the correlation between the return of the subject company and the return of the market portfolio. The mathematical procedure Damodaran used was to divide the "normal" beta by the coefficient of correlation. The result is the quotient of the standard deviation of the subject company divided by the standard deviation of the market portfolio, which Damodaran called "total beta."

Butler and Pinkerton recognized the capability of Damodaran´s TB to solve a serious problem facing professional appraisers. The central contribution of this approach is that it requires just one premium to be added to the risk-free rate, eliminating the need to estimate the premium for company-specific risks, a process that is often criticized as being susceptible to manipulation.[4] This and the simplicity of estimating TB were certainly the fundamental reasons that many valuation practitioners embraced this approach.

1 See Donald P. Wisehart, "Boston's Battle of the Beta," *Financial Valuation and Litigation Expert Journal*, no. 22 (December 2009/January 2010): 12.

2 See Aswath Damodaran, *Corporate Finance* (presentation, NYU Stern School of Business, New York, NY, 2001), 75, http://pages.stern.nyu.edu/~adamodar/pdfiles/country/CF2-day.pdf. (The first printed version of his approach can be found in the first edition of *Damodaran on Valuation* in 2002). See also, Aswath Damodaran, *Damodaran on Valuation: Security Analysis for Investment and Corporate Finance*, 2nd ed. (New York: John Wiley & Sons, 2006), 57.

3 Peter Butler and Keith Pinkerton, "Company-Specific Risk—A Different Paradigm: A New Benchmark," *Business Valuation Review* 25, no.1 (Spring 2006), 22–28.

4 For quotations from court cases, see Andrew M. Malec, "Using the Butler Pinkerton Calculator: A Case Study," *Business Valuation Update* 16, no. 12 (December 2010): 1.

As a "battle" needs enemies, there was also an ongoing critique.[5] We will not go into all the details of this critique, but it is important to recognize that although the TB concept may seem plausible, there is no closed formal derivation that can serve as a theoretical foundation comparable to the CAPM for public companies. As noted above, this is a common attribute of approaches for valuing private companies and, therefore, we should look at whether there are better alternatives. At least concerning the competing build-up models, TB proponents need not be concerned. As Donald Wisehart observed in his Beta-Battle report at the 2009 ASA conference:

> It was interesting to note that all panelists were in agreement that not only does Tβ violate CAPM, but the build-up "modified CAPM" that most valuators use today violates CAPM as well.[6]

We agree. When someone uses more risk premia, this is not a "modification" but a contradiction of the CAPM.[7] But what then? Do we have a rational concept to check the TB approach? Perhaps the most prominent proverb of German appraisers is "valuing means comparing." If we accept this and look at the basics of modern investment theory, we see that the efficient frontier for rates of return is the capital market line (CML), the region where you cannot gain a greater mean of return without accepting a greater standard deviation (or "volatility" or "dispersion"). This line is open to all investors, who can combine the market portfolio with a holding of riskless bonds or use debt to leverage the investment.[8] Thus, for each individual return volatility and the corresponding TB, the CML defines the efficient mean rate of return as the opportunity cost of the investment.

This insight is essential for a number of contributions to the literature that are at the periphery of the TB debate (because they focus on different purposes). These contributions do not use the term total beta, although they undoubtedly apply the concept.[9] The clearest expression of this rationale comes from Modigliani and Modigliani:

> We rely on a well-known proposition in the field of finance. Given any portfolio I, with total return r_i and dispersion σ_i, it is possible to construct a new version of that portfolio having any desired level of dispersion (risk). This can be accomplished by a financial operation called levering or unlevering the original portfolio.[10]

Under these circumstances, the discount rate or the opportunity cost of capital can easily be determined by applying a version of "pricing by duplication," or "pricing by replication,"[11] if one uses the market as the "original" portfolio or, as Kerins, Smith, and Smith formulated: "The approach of estimating cost of capital assumes that the entrepreneur can forego the venture and duplicate total portfolio risk by leveraging an investment in the market."[12]

Despite some differences, most of the recent German literature on TB follows this approach.[13] In contrast, there seems to be just one article in the U.S. TB debate that refers to the leveraged market portfolio. Richard Conn tried

5 See, e.g., Larry J. Kasper, "The Butler Pinkerton Model for Company-Specific Risk Premium—A Critique," *Business Valuation Review* 27, no. 4 (Winter 2008): 233–243; Larry J. Kasper, " Fallacies of the Butler Pinkerton Model and the Diversification Argument," *The Value Examiner* (January/February 2010): 8–20; Sarah von Helfenstein, "Revisiting Total Beta," *Business Valuation Review* 28, no. 4 (Winter 2009): 201–223; Richard R. Conn, "A Critique of Total Cost of Equity: Why TCOE Results May Not Be Defensible," *The Value Examiner* (January/February 2011): 9–19.

6 Wisehart, "Boston's Battle of the Beta," 13.

7 See, e.g., Christian Gilles and Stephen F. LeRoy, "On the Arbitrage Pricing Theory," *Economic Theory* 1, no. 3 (July 1991): 213–229 (a devastating critique of APT as an alternative to CAPM); Margaret Bray, *The Arbitrage Pricing Theory is Not Robust 1: Variance Matrices and Portfolio Theory in Pictures* (London School of Economics, Financial Markets Group Discussion Paper, 1994); Lutz Kruschwitz and Andreas Löffler, "Ross' APT Ist Gescheitert. Was Nun?" ("Ross's APT has Failed. What Now?)," *Zeitschrift für Betriebswirtschaftliche Forschung* 49, no. 7/8 (1997): 644–651.

8 For practical reasons, one must always assume that the theoretical world market portfolio is sufficiently represented by a market index. Criticism concerning this assumption is also opposed to the use of the CAPM for real-world valuations and will not be discussed here.

9 See Robert C. Camp and Arthur A. Eubank, Jr., "The Beta Quotient," *Journal of Portfolio Management* 7, no. 4 (Summer 1981), 53–58; Frank Kerins, Janet Kiholm Smith, and Richard Smith, "Opportunity Cost of Capital for Venture Capital Investors and Entrepreneurs," *Journal of Financial and Quantitative Analysis* 39, no. 2 (June 2004): 385–405; Daniel L. McConaughy and Vincent Covrig, "Owner's Lack of Diversification and the Cost of Equity Capital for a Closely Held Firm," *Business Valuation Review* 26, no. 4 (Winter 2007): 115–120. The role of the last article is not entirely clear. It did not use the term "total beta" at all and did not quote Damodaran or Butler and Pinkerton, even though the first Butler and Pinkerton article had already been published in the same journal in 2006 (see n. 3) and Covrig and McConaughy wrote a comment on the controversy between Butler and Pinkerton, and Kasper, in the winter 2008 issue of *Business Valuation Review* (see http://www.bvmarketdata.com/pdf/BPMAcademicCommentary.pdf).

10 Franco Modigliani and Leah Modigliani, "Risk-Adjusted Performance," *Journal of Portfolio Management* 23, no. 2 (Winter 1997): 47.

11 See, e.g., Klaus Spremann, *Valuation* (Munich: De Gruyter Oldenbourg, 2004), 261; Werner Gleißner and Marco Wolfrum, "Eigenkapitalkosten und die Bewertung nicht börsennotierter Unternehmen: Relevanz von Diversifikationsgrad und Risikomaß" (Cost of Equity and the Valuation of Unlisted Companies: Relevance of Degree of Diversification and Measure of Risk), *Finanz Betrieb* 10 (2008): 602–614, http://www.werner-gleissner.de/site/publikationen/WernerGleissner_Bewertung-nicht-boersennotierter-Unternehmen.pdf.

12 Kerins, Smith, and Smith, "Opportunity Cost of Capital," 387.

13 See, e.g., Ulrich Balz and Heinz-Gerd Bordemann, "Ermittlung von Eigenkapitalkosten zur Unternehmensbewertung mittelständischer Unternehmen mit Hilfe des CAPM" ("Determination of the Cost of Equity for Valuing Midsize Companies on the Basis of the CAPM"), *Finanz Betrieb* 9 (2007): 737–743; Gleißner and Wolfrum (see n. 11); Hans Dirrigl, "Unternehmensbewertung für Zwecke der Steuerbemessung im Spannungsfeld von Individualisierung und Kapitalmarkttheorie—Ein aktuelles Problem vor dem Hintergrund der Erbschaftsteuerreform" ("Company Valuation for Tax Assessment Purposes in the Area of Conflict between Individualization and Capital Market Theory—A Current Problem Against the Background of Inheritance Tax Reform"), in *Accounting, Taxation, and Corporate Governance: Essays in Honor of Franz W. Wagner on the Occasion of his 65th Birthday*, ed. Dirk Kiesewetter and Rainer Niemann, 2009, Part B, B-51, http://www.franz-w-wagner.de; Leonhard Knoll, "KMU-Bewertung: Kapitalmarktorientierte Risikoberücksichtigung ohne Börsennotiz und Diversifikation?" ("SME Valuation: Capital Market-Based Risk Consideration without Stock Exchange Listing and Diversification?"), *Österreichische Zeitschrift für Recht und Rechnungswesen* 20 (2010): 365–371.

to use this idea as a counterargument against TB.[14] The failure of this counterargument is best demonstrated by his own judgment on page 12 of his article: "Would he [a normally risk-averse investor] presume that the probability of Company B becoming insolvent and worthless was just as likely as every firm in the S&P 500 simultaneously failing?"

Conn did not recognize that a simultaneous failure of every firm is not necessary for a leveraged market portfolio to become worthless. It is enough that the index performs so badly that the investor cannot pay back the debt (including the interest) from the portfolio's returns. If the data and calculations are correct, the risk of this is indeed as high as the risk of insolvency of a single firm that the investor has acquired with his or her own equity.

This discussion shows that the replication approach provides a method for theoretically substantiating the TB approach. The replication approach can thus be used to derive valuation situations with perfect concentration and full diversification. The difference between CAPM and TB lies in the risk diversification possibilities.

Implementation Issues

In our view, another fundamental reason for TB´s success was the way Butler and Pinkerton proposed its implementation. First, they always stressed reliance on market comparisons—an important psychological point in a professional world where many appraisers use very expensive data sources, such as Bloomberg or Thomson Financial, that should not be useless for the new concept. Second, soon after their initial article, they constructed a new and elegant tool for estimating TB—the previously mentioned BPC. With that, the hurdle for potential users with some aversion to math and models was clearly eliminated.

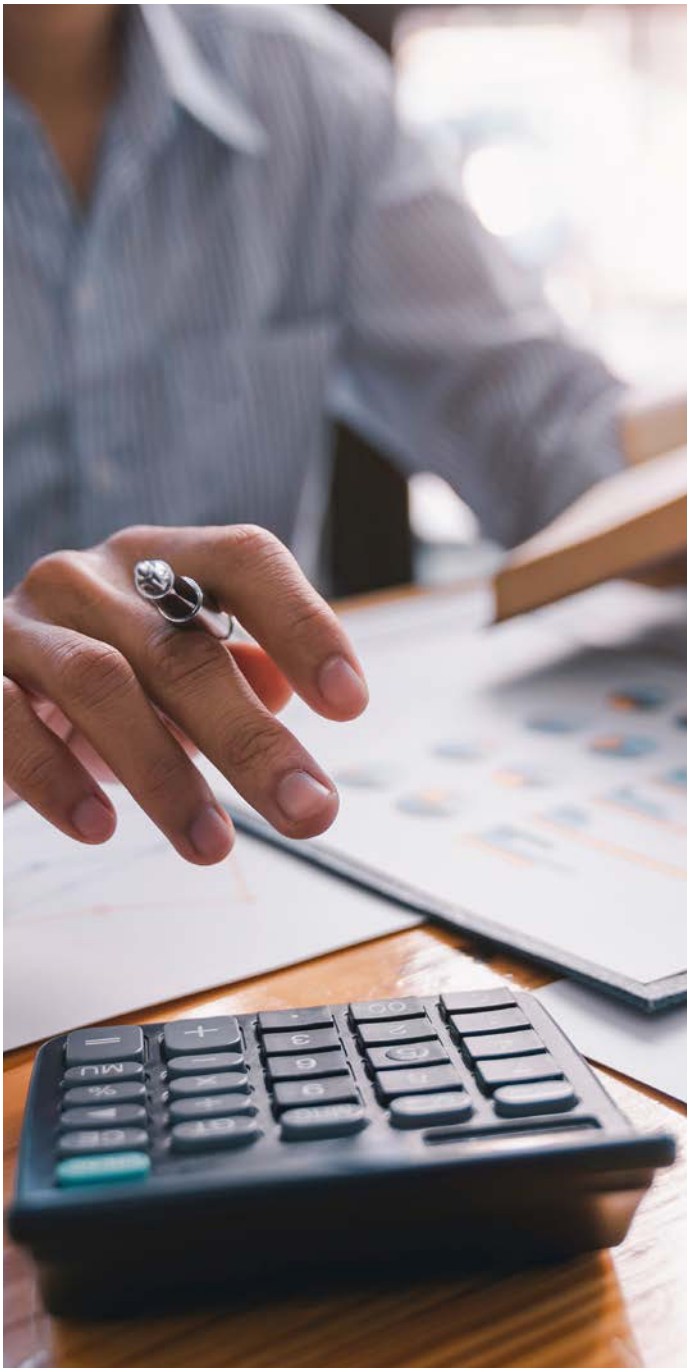
As with conceptual issues, this methodology was heavily criticized by some authors.[15] But while the critics challenged the specifics of the model, they did not question the general reliance on market comparisons. If you believe in peer-betas, then you can trust the TBs of comparable public corporations as well. If you do not believe in peer-betas, however,[16] you should look for an alternative. This seems to be simpler here

14 See Conn, "A Critique of Total Cost of Equity," 11–12.

15 See n. 5.

16 There are good reasons to be skeptical. See Leonhard Knoll, Jan Ehrhardt, and Florian Bohnet, "Kleines Beta—Kleines Bestimmtheitsmaß: Großes Problem?" ("Small Beta—Small Measure of Determination: Big Problem?"), *CFO Aktuell* 1 (2007): 210; Leonhard Knoll, "Äquivalenz zwischen signifikanten Werten des Beta-Faktors und des Bestimmtheitsmaßes" ("Equivalence between Significant Values of the Beta Factor and the Coefficient of Determination"), *Die Wirtschaftsprüfung* 63 (2010): 1106.

than in the estimation of CAPM because you only need the standard deviation of the valuation subject; there is no need for a correlation term that cannot be determined without an exchange listing. Nevertheless, things remain somewhat complex because the rate of return is the quotient of the monetary return divided by the value invested (minus one) and the value invested is unknown—it is the objective of our calculation.



To avoid this circular reasoning, you can change the procedure: instead of discounting the mean of the monetary return, you can discount the certainty equivalent of the cash flow by the rate of return of a risk-free investment. For the classical, one-period view, the transformation is straightforward:[17]

$$r_A = i + (r_M - i) \cdot \frac{\sigma_{\tilde{r}_A}}{\sigma_{\tilde{r}_M}} \leftrightarrow 1 + i = 1 + r_A - \frac{(r_M - i)}{\sigma_{\tilde{r}_M}} \cdot \sigma_{\tilde{r}_A} \leftrightarrow$$

$$V_A = \frac{V_A \cdot (1 + r_A) - \frac{(r_M - i)}{\sigma_{\tilde{r}_M}} \cdot V_A \cdot \sigma_{\tilde{r}_A}}{1 + i} \leftrightarrow$$

$$V_A = \frac{\overbrace{E[\tilde{e}_A] - \frac{(r_M - i)}{\sigma_{\tilde{r}_M}} \cdot \sigma[\tilde{e}_A]}^{\text{certainty equivalent}}}{1 + i}$$

Where:

i	=	Rate of return for a risk-free asset
$r_M = E[\tilde{r}_M]$	=	Mean of the market rate of return
$\sigma_{\tilde{r}_M}$	=	Standard deviation of the market rate of return
$r_A = E[\tilde{r}_A]$	=	Mean of the rate of return for a single investment in firm A
$\sigma_{\tilde{r}_A}$	=	Standard deviation of the rate of return for a single investment in firm A
V_A	=	Value of firm A in t = 0
$E[\tilde{e}_A]$	=	Mean of the monetary return (final value) of firm A (e.g., flow to equity)
$\sigma[\tilde{e}_A]$	=	Standard deviation of the monetary return (final value) of firm A

Applying this equation, the discount rate and, with it, TB, disappears. Nevertheless, we follow the TB approach precisely, but now we estimate not only the mean of the cash flow—as in every discounted cash flow (DCF)

17 See, e.g., Knoll (n. 13), 369; proof using a replication approach can be found in Werner Gleißner and Marco Wolfrum, "Cost of Capital and Valuation with Imperfect Diversification and Unsystematic Risks," Finexpert, 2009, http://www.werner-gleissner.de/site/publikationen/WernerGleissner_Cost-of-Capital-and-Valuation-with-imperfect-Diversification-and-unsystemativ-Risks.pdf. In the case of partial diversification, value additivity does not apply; we will not discuss the implications here. Some sources that use this concept do not show the algebra but just the result. See, e.g., McConaughy and Covrig, "Owner's Lack of Diversification," 117.

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valuation—but also its standard deviation. One can do this by using probability-weighted scenario values or by running simulations.[18] This may seem unusual, but remember that different scenarios should be used for the estimation of cash flows and simulation analysis has been proposed for the comparison of investments since the 1960s.[19]

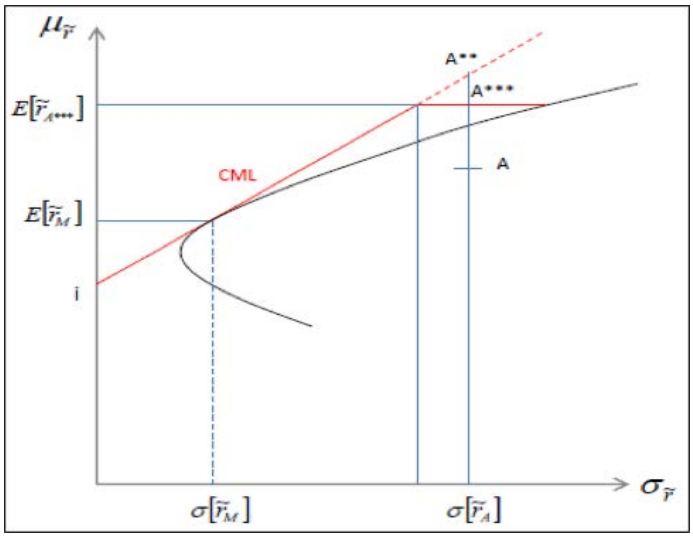
Compared with the estimation problems of build-up models, particularly with regard to the "company-specific risk premium," both TB alternatives are certainly a lesser evil. Thus, the choice is between these alternatives, and here lies a clear difference between the U.S. and German literature.[20] Not surprisingly, we prefer the use of the cash flow deviation instead of estimating TB with comparable public corporations. But even this procedure can be avoided in many cases, as we show below.

Heuristics

In light of the above discussion, it is interesting to look at the normal level of discount rates determined by applying TB. This is easily accomplished when you use the certainty equivalent alternative because, after having calculated the value of a firm, you can easily derive the discount rate. Whether determined by this method or by direct estimation with reference to comparable public corporations, the discount rate often reaches 20 percent or even more. At least in Germany, this has provoked heavy criticism, with one commentator describing it as "pathological risk aversion."[21] Looking at available economic (not book) returns in reality, we agree with this criticism,[22] but there is another problem with such high rates of return under the TB approach that requires some consideration.

The level of debt for acquiring shares is not unlimited. The maximum amount of debt is either a fixed percentage of the total investment (most common in the German-speaking countries) or corresponds to the usual rating with respect to the probability of default. When this limit is reached, it is not possible to increase the expected rate of return with further borrowing. Thus, the relevant frontier has a turning point: to the right of the limit, the CML is no longer relevant, but is a parallel to the sigma-axis (see Figure 1).

Figure 1: CML and the Maximum Amount of Debt



Point A** in the figure shows the combination of μ and σ that is equal for an equity-financed investment in firm A and a correspondingly leveraged investment in the market portfolio. Regrettably, this point is not feasible because it is to the right of the maximum amount of debt. Therefore, point A*** on the parallel to the sigma-axis determines the discount rate.

Critics of TB will argue that the μ-σ combination of this point is not efficient: the maximum leveraged investment in the market portfolio yields the same expected return with a smaller standard deviation. Nevertheless, this situation represents the empirical norm for private companies. Can we find a rationale for it? Yes, we can. Up to this point, we have considered the missing diversification as a disadvantage, but it also has a positive side: in many cases, the concentration on one investment corresponds with more influence on business policy and better incentives to preserve the core investment. This is true not only if the owner is the manager, but also if the owner(s) can control management because of their high level of equity investment in the firm. For example, they can urge management to abandon investments if they fear a worsening of economic conditions or find other ways to limit the risk of entrepreneurial decisions.[23]

18 See McConaughy and Covrig, "Owner's Lack of Diversification," 117.

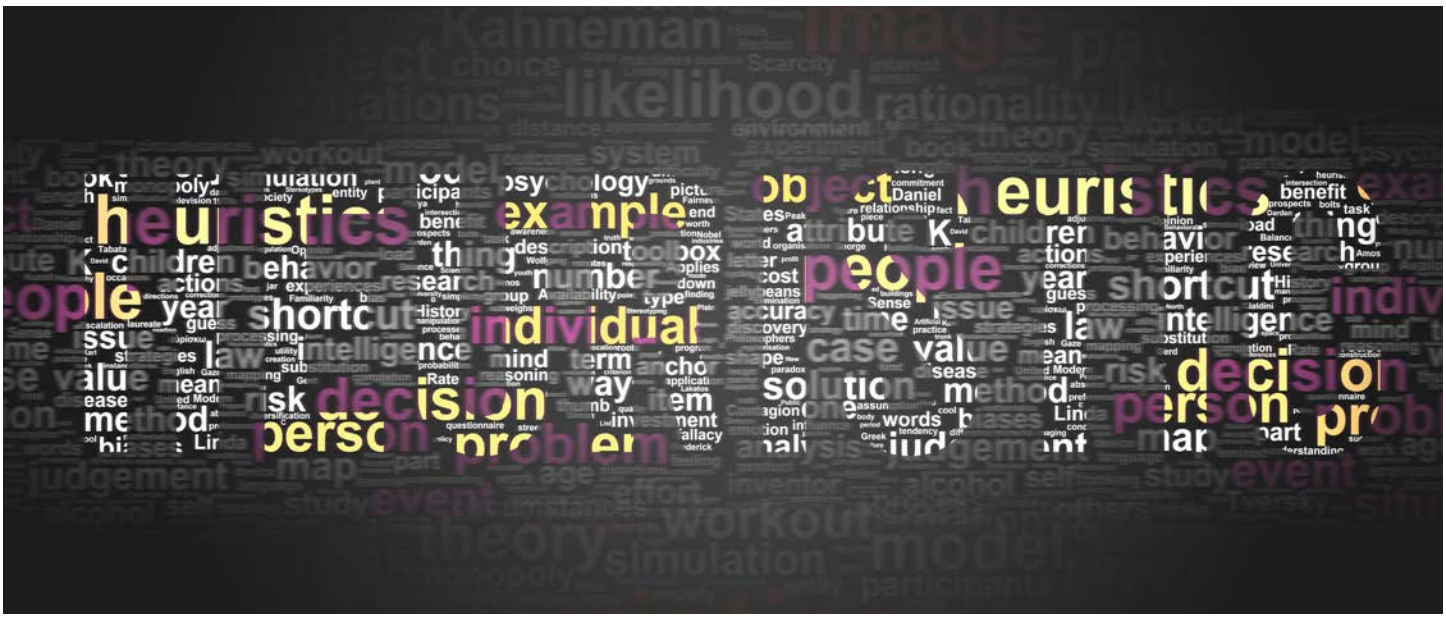
19 See David B. Hertz, "Risk Analysis in Capital Investment," *Harvard Business Review* 42, no. 1 (January-February 1964): 95.

20 Interestingly, Kerins, Smith, and Smith combine the certainty equivalent formulation with the use of data of comparable public firms. Kerins, Smith, and Smith, "Opportunity Cost of Capital," 392. To our knowledge, McConaughy and Covrig are the only U.S. authors to really implement the relevant certainty equivalent concept in a concrete (exemplary) valuation. McConaughy and Covrig, "Owner's Lack of Diversification" (see n. 9). However, that article did not play a role in the U.S. discussion concerning TB.

21 See Gernot W. Zeidler, "Die Anwendbarkeit von IDW S 1 auf kleine und mittlere Unternehmen," (The Applicability of IDW S 1 to Small and Medium-Sized Enterprises"), in *Besonderheiten der Bewertung von Unternehmensteilen sowie von kleinen und mittleren Unternehmen (Special Features of the Valuation of Parts of Companies and Small and Medium-Sized Enterprises)*, ed. Jörg Baetge and Hans-Jürgen Kirsch (Düsseldorf, Ger.: IDW-Verlag, 2005), 48.

22 In the U.S. as well, there are some doubts about whether the resulting impact on value is plausible. For example, looking at an empirical measure of using TB instead of beta, Lee stated: "The median cut in value is 69.5%. This drop is enormous." M. Mark Lee, "Using Total Beta and the Butler Pinkerton Calculator to Solve the CAPM Credibility Problem," *Business Valuation Review* 29, no. 3 (Fall 2010): 77.

23 See Michael Keller and Bruno Hohmann, "Besonderheiten Bei der Bewertung von KMU" ("Special Features in the Valuation of SMEs"), in Unternehmensbewertung: Moderne Instrumente und Lösungsansätze, ed. Frank Richter and Christian Timmreck (Stuttgart, Ger.: Schäffer-Poeschel, 2004) 208.



The value of this flexibility[24] corresponds in general to the concept of real options, which normally are not fully accounted for in DCF valuations.[25] For this and other reasons, investors are often prepared to pay a control premium, although to acquire a controlling interest itself contradicts perfect diversification and should, therefore, lead to a lower price than that calculated using the CAPM. Recognizing this—and the fact that, in most cases, A** is not very far from the turning point (sometimes it is even to the left of it)—there is little error in using the expected rate of return corresponding to that point.

This discussion leads us to another interesting question: when determining the upper borrowing limit, do we need to distinguish between a fully diversified investor and a non-diversified investor? The rational decision of an investor in the CAPM is to hold a fully diversified portfolio. Adopting the assumptions of the CAPM—i.e., perfectly diversified investors—d is the correlation between $\tilde{e}_A$ and the market portfolio, which also flows into the calculation of the CAPM beta. A robust heuristic for a "medium" diversification of investors is $d = 0.5$. In the TB approach, we have the situation of a completely non-diversified investor—that is, $d = 1$.

With simulation-based valuation, there are more degrees of freedom with regard to diversification. Here, the CAPM and the TB approach are the two extremes within which different degrees of diversification are possible (see equation below). The degree of diversification depends on the investor's specific asset situation and portfolio structure, and can be calculated individually.

$$V_A = \frac{E[\tilde{e}_A] - \frac{(r_M - i)}{\sigma_{\tilde{r}_M}} \cdot \sigma[\tilde{e}_A] \cdot d}{1+i}$$

Where d = diversification factor

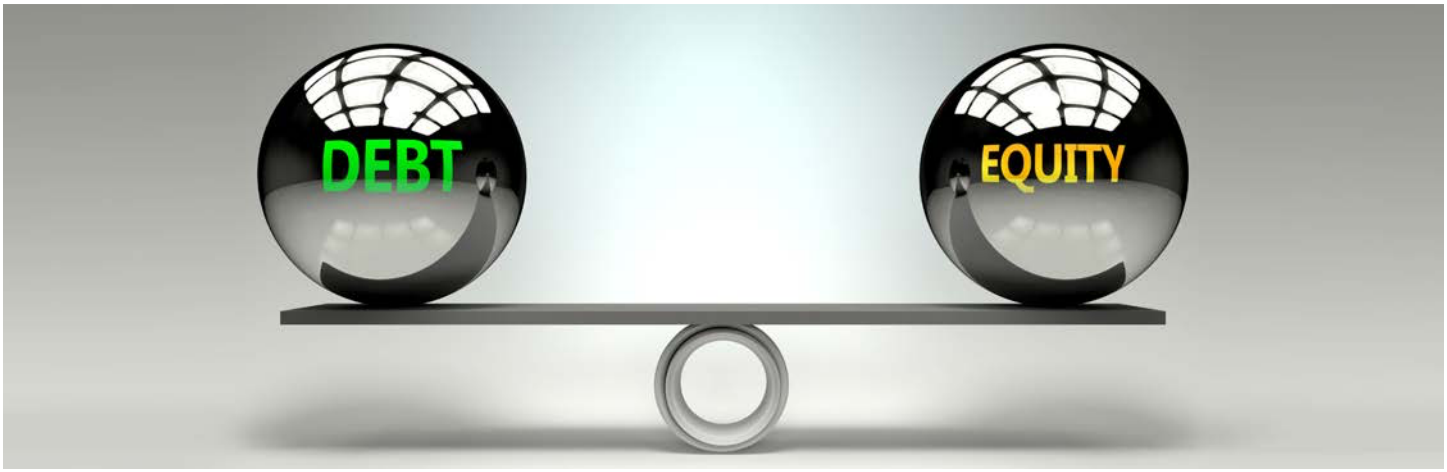
There are various methods for calculating the diversification factor. Further research is being conducted on this topic in connection with simulation-based valuation. It is possible to determine the diversification factor using a copula function[26] or a simulation calculation taking into account the investor's portfolio.[27] The insolvency risk associated with diversification depends, in turn, on how risky the planned cash flows of the individual assets are and how correlated they are with each other.

24 See Thomas E. Copeland, J. Fred Weston, and Kuldeep Shastri, *Financial Theory and Corporate Policy*, 4th ed. (Boston: Addison Wesley, 2005), ch. 9; Damodaran, *Damodaran on Valuation,* 433; Sheridan Titman and John D. Martin, *Valuation: The Art and Science of Corporate Investment Decisions* (Boston: Addison Wesley, 2008), 95.

25 See Copeland, Weston, and Shastri, *Financial Theory and Corporate Policy*, ch. 9. There is reasonable doubt as to whether the value effect can be quantified analogously to financial options. See Lutz Kruschwitz, *Investitionsrechnung (Capital Budgeting)*, 11th ed. (Munich: De Gruyter Oldenbourg, 2007), 461–465.

26 See Pascal Bruhn and Dietmar Ernst, "Assessing the Risk Characteristics of the Cryptocurrency Market: A GARCH-EVT-Copula Approach," *Journal of Risk and Financial Management* 15, no. 8 (August 2022): 1–28.

27 Cf. Gleißner and Wolfrum, "Eigenkapitalkosten und die Bewertung nicht börsennotierter Unternehmen" (see n. 11); Werner Gleißner, *Grundlagen des Risikomanagements*, 4th ed. (Munich: Vahlen, 2022), 521ff.



By following the heuristics based on these insights and applying them to determine the discount rate, there is no need to estimate the TB itself.

The Rule of Thumb

The first heuristic is a very simple one. It applies when the amount of debt is limited by a percentage of the total investment in shares. If E is the equity invested, D is the debt (a share Lombard credit), and B is the debt limit as a percentage of the total investment in the market portfolio, the rule is:

$$D = B \cdot (E + D) \leftrightarrow D = \frac{B}{1-B} \cdot E$$

For this leverage, the expected rate of return is 1/(1-B) times the market risk premium (= r_m-i), higher than the rate of return for a risk-free asset. Often, B lies in the 50–60 percent range. For 50 percent, you must add two times the market risk premium to the return of the risk-free asset to get the discount rate. Thus, all that is required to apply the heuristic are the values for r_m and i.[28]

The Rating Related Model

Currently, the general procedure for limiting the amount of debt follows the rules of rating. Here, the loan amount is determined on the basis of the accepted probability of default.

For the estimation procedure concerning the leveraged investment in the market portfolio, the mean and the standard deviation of its rate of return are required. If r_d = the contracted rate of return for the share Lombard credit,[29] q_p = p–% quantile of the standardized normal distribution, and we make some simplifying assumptions, the discount rate is calculated as follows:[30]

$$r_A = \frac{r_m \cdot (1+r_d) - i \cdot (1+r_m + q_p \cdot \sigma_m)}{r_d - (r_m + q_p \cdot \sigma_m)}$$

For $r_d = i$, the formula can be further simplified as follows:

$$r_A = i + \frac{r_m - i}{-(r_m + q_p \cdot \sigma_m)}$$

Obviously, the discount rate depends on the accepted probability of default and the corresponding value for q_p. The denominator expresses the equity portion of the leveraged portfolio (equity requirement as a percentage of the investment) that is necessary for a normal distribution of returns, so that the probability of default just reaches p. It is important to note that by using the method of "incomplete replication" and risk-value models, it is possible to derive these results as a specific case of a general approach for the valuation of risky cash flows.[31]

The simplest way to assess the probability of insolvency/default is to use exogenous credit ratings (e.g., from a rating agency). An estimate of the probability of default can be directly derived from a published rating, as shown in Table 1.

28 See Knoll, "KMU-Bewertung," 369, for further discussion of this result.

29 At this point, one may feel uncomfortable with the fact that leverage of the market portfolio is not determined with the risk-free interest rate. Nevertheless, the actual difference is not important, if one regards ratings with low accepted probabilities of default and remembers that most of the increase in the debt interest compensates for it. In a competitive market, the remaining difference in the form of a risk premium should be low (see our example below where we assume 0.5 percent).

30 See Werner Gleißner and Leonhard Knoll, "Konsistente Bewertung von Eigen und Fremdkapital durch ratingabhängige Risikozuschläge: ein Vorschlag für KMU" ("Consistent Valuation of Equity and Debt through Rating-Based Risk Premiums: A Proposal for SMEs"), *Betriebs-Berater* 37 (2011): 2283; Werner Gleißner, "Der Einfluss der Insolvenzwahrscheinlichkeit (Rating) auf den Unternehmenswert und die Eigenkapitalkosten" ("The Influence of the Probability of Insolvency (Rating) on the Company Value and the Cost of Equity Capital"), *Corporate Finance Biz* 4 (2011), 249 (with a formulation for the case that the bank does not include the interest payments in the VaR calculation).

31 Cf. Gregor Dorfleitner and Werner Gleißner, "Valuing Streams of Risky Cashflows with Risk-Value Models," *Journal of Risk* 20, no. 3 (2018): 1–27; Werner Gleißner, "Cost of Capital and Probability of Default in Value-Based Risk Management," *Management Research Review* 42, no. 11 (2019): 1243–1258.

Table 1: Average Cumulative Probability of Insolvency/Default from 1981 to 2020 (in percent)

Rating	Time Horizon							
	1	2	3	4	5	6	7	8
AAA	0.00	0.03	0.13	0.24	0.34	0.45	0.51	0.59
AA	0.02	0.06	0.11	0.21	0.30	0.41	0.49	0.56
A	0.05	0.13	0.22	0.33	0.46	0.60	0.76	0.90
BBB	0.16	0.43	0.75	1.14	1.54	1.94	2.27	2.61
BB	0.63	1.93	3.46	4.99	6.43	7.75	8.89	9.90
B	3.34	7.80	11.75	14.89	17.35	19.36	20.99	22.31
CCC/C	28.30	38.33	43.42	46.36	48.58	49.61	50.75	51.49
Rating	Time Horizon							
	9	10	11	12	13	14	15	
AAA	0.64	0.70	0.72	0.75	0.78	0.84	0.90	
AA	0.63	0.70	0.76	0.82	0.88	0.93	0.99	
A	1.05	1.20	1.34	1.46	1.59	1.71	1.84	
BBB	2.93	3.24	3.55	3.80	4.03	4.28	4.54	
BB	10.82	11.64	12.33	12.99	13.59	14.09	14.65	
B	23.50	24.62	25.58	26.31	26.99	27.63	28.24	
CCC/C	52.16	52.76	53.21	53.68	54.23	54.69	54.76	

We can now calculate the discount rate in addition to the rating or the probability of default (p). This depends on the profitability, the leverage-ratio, and the total volume of risk of the firm.

If we assume the following:

- Rating = BBB-
- p = 0.5% and, therefore, $q_p = -2.576$
- r_d = 5% and i = 4%
- σ_m = 20% and r_m^e = 8%

Then:

$$r_A = \frac{r_m \cdot (1+r_d) - i \cdot (1+r_m + q_p \cdot \sigma_m)}{r_d - (r_m + q_p \cdot \sigma_m)} = \frac{8\% \cdot (1+5\%) - 4\% \cdot (1+8\% - 2.576 \cdot 20\%)}{5\% - (8\% - 2.576 \cdot 20\%)} = 12.7\%$$

It is easy to calculate the cost of capital for different ratings if we assume that $r_d = i + p + r_Z$ with a simplifying assumption of a constant premium, r_Z = 0.5%, charged by the creditor:

Rating	p	q_p	r_A
AAA	0.00001%	-5.20	8.2%
BBB	0.25%	-2.81	11.9%
BB	0.95%	-2.35	13.5%
B	4.7%	-1.67	16.6%

For normal rating levels, the result does not differ much from the rule of thumb and, again, there is no need for capital market data or cash flow parameters of single firms.

The Logical Starting Point for Professional Judgment

Both heuristics lead to a discount rate that is independent of the valuation subject. This may seem questionable, but it brings professional judgment back to a logical starting point: because the general investment opportunity is fixed by the kinked μ-σ frontier, the appraiser can concentrate on idiosyncratic deviations from the heuristic result. Looking at the derivation of the heuristics, these deviations must be substantial and should be clearly formulated to convince a third person (e.g., a judge) why one should use a higher or smaller discount rate than the heuristic result.[32] Therefore, there is much less leeway than in the determination of the discount rate using conventional build-up models, where the number of risk factors, their correlation, and their measurement already span a range of possible outcomes before the professional judgment concerning company-specific risks comes into play.

The methodology described in this article reveals the impact of professional judgment, which can then be challenged directly by any recipient of the valuation. In other words, the heuristics do not eliminate professional judgment; rather, they clearly disclose its role in arriving at the valuation result.

32 See Knoll, "KMU-Bewertung: Kapitalmarktorientierte Risikoberücksichtigung ohne Börsennotiz und Diversifikation?," 370.

The U.S. debate on the valuation of private companies and the corresponding discussions in the rest of the world are far from finished.

Final Remarks

We do not know whether our views on TB will gain acceptance in the U.S. debate. As we wrote in the introduction, modesty is needed because the theoretical ground for the valuation of private companies is far less stable than for publicly listed corporations. Nevertheless, market imperfections mean that the CAPM-based DCF valuation does not do justice to many valuation situations.

The total beta approach, which can be axiomatically justified with the replication approach ("imperfect replication"), represents an interesting alternative. It allows for the capture of special features of the valuation subject that are not captured by a CAPM-based valuation. It is appropriate if the valuation subject has no risk diversification options, especially if virtually all of its assets are tied up in the company being valued. With the additional concept explained in the article—the rating-related model (called the "risk coverage approach" in German literature)—such risk diversification options are dispensed with, as with the TB approach, and existing rating and financing restrictions are also taken into account. With this model, which can be derived from the same theoretical basis, discount rates can be determined depending on the rating or insolvency probability and the findings from a risk analysis can be taken into account.

It is important to recognize that there is a certain degree of conceptual unclarity and to search for the least of several evils. We believe there is an argument to be made that TB is the least of all evils, especially compared to most build-up models. Personally, we prefer to use the certainty equivalent version of the TB concept. In most cases, we feel comfortable using the heuristics; given the uncertainty of the valuation process, striving for perfection is futile. Of course, the appraiser may need to make small corrections if the company to be valued is quite safe/risky or the buyer/seller has significant funds invested outside of it, but (at least outside of litigation) professional judgment will usually suffice. Again, the issue is much less significant in comparison to the build-up models, whose leeway in the determination of the company-specific risk premium was one of the reasons for the success of the TB approach. With currently available valuation theory based on replication models, it can be shown that the valuation results of the TB approach, the CAPM, and the rating-related model (which derives discount rates from rating and risk analysis) are special cases of valuations that differ in assumptions about financing restrictions and risk diversification options.

Finally, the approach we suggest is a shortcut and we expect further progress in the field. The U.S. debate on the valuation of private companies and the corresponding discussions in the rest of the world are far from finished. VE

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